

# EIGENVECTOR-BASED RELATIONAL MOTIF DISCOVERY

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## ABSTRACT

The development of novel analytical tools to investigate the structure of music works is central in current music information retrieval research. In particular, music summarization aims at finding the most representative parts of a music piece (motifs) that can be exploited for an efficient music database indexing system. Here we present a novel approach for motif discovery in music pieces based on an eigenvector method. Scores are segmented into a network of bars and then ranked depending on their centrality. Bars with higher centrality are more likely to be relevant for music summarization. Results on the corpus of J.S.Bach's 2-part Inventions demonstrate the effectiveness of the method and suggest that different musical metrics might be more suitable than others for different applications.

## 1. INTRODUCTION

Listening to music and perceiving its structure is a relatively easy task for humans, even for listeners without formal musical training. However, building computational models to simulate this process is a hard problem. On the other hand, the problem of automatically identifying relevant characteristic motifs and efficiently store and retrieve the digital content has become an important issue as digital collections are increasing in number and size more or less everywhere.

Notwithstanding the conspicuousness of the literature, current approaches seem to rely just on the *repetition* paradigm [20] [8], assigning higher scores to recurring equivalent melodic and harmonic patterns [11]. Recently reported approaches to melodic clustering based on string compression [10], motivic topologies [18], graph distance [21] and paradigmatic analysis [19] have been used to select relevant subsequences among highly repeated ones by heuristic criteria [15] [1]. However, this approach is not completely satisfying as the repetition paradigm can provide just a first approximation of the perceptual ranking

mechanism [3] and produces too many false positives sharing the same repetition rates.

Moreover, the repetition paradigm, in order to be applied, needs by no means a precise definition of "*varied repetition*", a concept not easy to define. Of course, it has to include standard music transformation, but it is very difficult to adopt a simple two-valued logic (this is a repetition and this is not) in this context, where a more fuzzy approach seems to better address such a problem.

Sometime repetitions may even lead to evident mistakes, as it might happen that highly repeated patterns turn to be totally irrelevant from a musicological point of view. In fact cases occur where the most repeated pattern in the whole composition is an ornament, like a trill. This is to show that the repetition paradigm is not sufficient in itself to identify relevant themes but it needs some heuristics to select among relevant and irrelevant patterns.

Here we present an alternative ranking method based on connections instead of repetitions. We show that a distance distribution on a graph of note subsequences induced by music similarity measures generates a ranking real eigenvector whose components reflect the actual relevance of motives. False positives of the repetition paradigm turned out to be less connected nodes of the graph due to their higher degree of dissimilarity with relevant motives.

Our results show how higher indexes of connection, or "centrality", are more likely to perform better than higher repetition rates in motif discovery, with no additional assumptions on the particular nature of the sequence or the adopted similarity measure.

## 2. RELATED WORKS

Music segmentation is usually realized through musicological analysis by human experts and, at the moment, automatic segmentation is a difficult task without human intervention. The supposed music themes have often to undergo a hand-made musicological evaluation, aimed at recognizing their expected relevance and completeness of results. As a matter of fact, an automatic process could extract a musical theme which is too long, or too short, or simply irrelevant. That's why a human feedback is still required in order to obtain high-quality results.

We present here an overview of current approaches based on different musical assumptions. We start this section with a general overview of the literature. Then we intro-

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duce harmony related approaches, with a focus to reductionistic ones. Finally we introduce topology-based models, which share much more similarities than others with our approach. All those methods makes use of the repetition paradigm.

## 2.1 General approaches

Lartillot [15] [16] defined a musical pattern discovery system motivated by human listening strategies. Pitch intervals are used together with duration ratios to recognize identical or similar note pairs, which in turn are combined to construct similar patterns. Pattern selection is guided by paradigmatic aspects and overlaps of segments are allowed.

Cambouropoulos [6], on the other hand, proposed methods to divide given musical pieces into mostly non-overlapping segments. A prominence value is calculated for each melody based on the number of exact occurrences of non-overlapping melodies. Prominence values of melodies are used to determine the boundaries of the segments [7]. He also developed methods to recognize variations of filling and thinning (through note insertion and deletion) into the original melody. Cambouropoulos and Widmer [9] proposed methods to construct melodic clusters depending on the melodic and rhythmic features of the given segments. Basically, similarities of these features up to a particular threshold are used to determine the clusters. High computational costs of this method make applications to long pieces difficult.

## 2.2 Tonal harmony-based approaches

Tonal harmony based approaches exploit particular harmonic patterns (such as tonic-subdominant-dominant-tonic), melodic movements (e.g. sensible-tonic), and some rhythmical punctuation features (pauses, long-duration notes, ...) for a definition of a commonly accepted semantic in many ages and cultures.

These approaches typically lead towards score reductions (see Figure 1), made possible by taking advantage of additional musicological information related to the piece and assigning different level of relevance to the notes of a melody. For example one may choose to assign higher importance to the stressed notes inside a bar [22]. In other words, the goal of comparing two melodic sequences is achieved by reducing musical information into some “primitive types” and comparing the reduced fragments by means of suitable metrics.

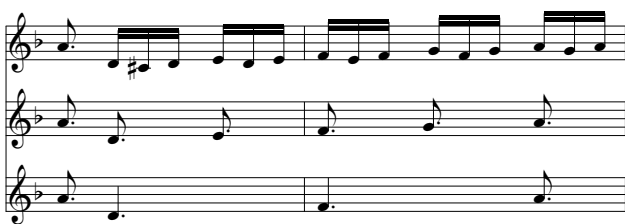


Figure 1. J.S. Bach, BWV 1080: Score reductions.

A very interesting reductionistic approach to music analysis has been attempted by Fred Lerdahl and Ray Jackendoff. Lerdahl and Jackendoff [17] research was oriented towards a formal description of the musical intuitions of a listener who is experienced in a musical idiom. Their purpose was the development of a formal grammar which could be used to analyze any tonal composition.

The study of these mechanisms allows the construction of a grammar able to describe the fundamental rules followed by human mind in the recognition of the underlying structures of a musical piece.

## 2.3 Topological approaches

Mazzola and Buteau [5] proposed a general theoretical framework for the paradigmatic analysis of the melodic structures. The main idea is that a paradigmatic approach can be turned into a topological approach. They consider not only consecutive tone sequences, but allow any subset of the ambient melody to carry a melodic shape (such as rigid shape, diastematic shape, etc.). The mathematical construction is very complex and, as for the motif selection process, it relies on the repetition paradigm.

The method proposed by Adiloglu, Noll and Obermayer in [1] does not take into account the harmonic structure of a piece and is based just on similarities of melodies and on the concept of similarity neighborhood. Melodies are considered as pure pitch sequences, excluding rests and rhythmical information.

A monophonic piece is considered to be a single melody  $M$ , i.e. they reduce the piece to its melodic surface. Similarly, a polyphonic piece is considered to be the list  $M = (M_i)_{i=1, \dots, N}$  of its voices  $M_i$ . The next step is to model a number of different melodic transformations, such as transpositions, inversions and retrogradations and to provide an effective similarity measure based on cross-correlation between melodic fragments that takes into account these transformations. They utilize a mathematical distance measure to recognize melodic similarity and the equivalence classes that makes use of the concept of *neighbourhood* to define a set of similar melodies.

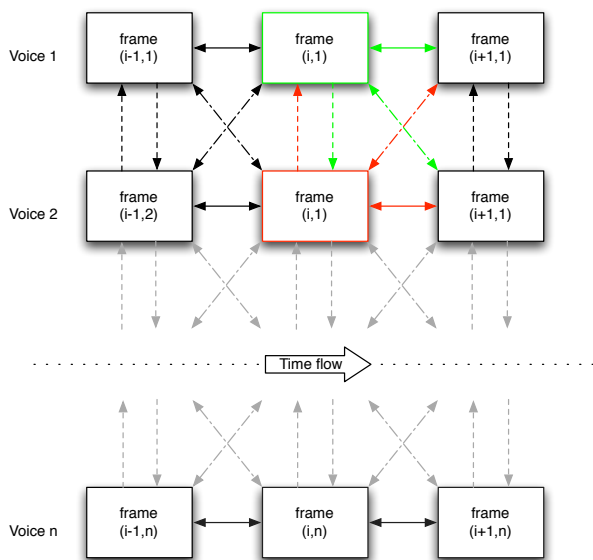
Following the repetition paradigm stated by Cambouropoulos in [7] they define a prominence value to each melody based on the number of occurrences, and on the length of the melody. The only difference is that they allow also melody overlapping. In the end, the significance of a melody  $m$  of length  $n$  within a given piece  $M$  is the normalized cardinality of the similarity neighbourhood set of the given melody. If two melodies appear equal number of times, the longer melody is more significant than the shorter one.

In [1] the complete collection of the Two-part Inventions by J. S. Bach is used to evaluate the method, and this will be also our choice in section 4.

## 3. THE RELATIONAL MODEL

As stated in Section 2, current methods rely on the repetition paradigm. Our point of view can be synthesized in the following points:

1. We consider a music piece as a network graph of segments,
2. we do take into account both melodic and rhythmical structures of segments
3. we do not consider harmony, as it is too much related to tonality.



**Figure 2.** A representation of the (first-order) network of frames.

A single frame may represent, for instance, a bar or a specific voice within a bar like in Fig. 2, but also more general segments of the piece. We do not take into account here the problem of windowing as the method is basically independent from any specific segmentation of the piece. What we provide here is a different point of view which, like the repetition paradigm, can be applied in principle to any specific segmentation.

### 3.1 Score representation

The natural consequence is that a music piece can be looked at like a complete graph  $K_n$ . In graph theory, a complete graph is a simple graph where an edge connects every pair of distinct vertices. The complete graph on  $n$  vertices has  $n(n-1)/2$  edges and is a regular graph of degree  $n-1$ .

In this representation, score segments correspond to graph nodes and the similarity between couples of segments correspond to edge weights. This approach can be better explained if we think to a score like a network graph of “pages”, so we can establish a parallelism between score segments ranking and the World Wide Web ranking process as originally depicted in [4] by S. Brin and L. Page.

As stated before, the problem of windowing is partly overcome in the network concept as it does not strongly affect the model. In fact by using undersized windows we normally get just more detailed results. In our experiments

(see Section 4) we decided to adopt a one-bar length window, as we considered metric information relevant to music segmentation, avoiding any form of overlapping. In fact it turned out that if the metric information is taken into account, overlapping windows are not relevant in a relational model, as they can lead to inaccurate motif discovery, due to overrates given to highly self-similar segments.

### 3.2 Metric weights

As stated above, graph nodes correspond to score segments. The next issue is the definition of a suitable concept of distance between segments. This should be apparently at the very heart of the method, and in a sense it is. Every time there is a similarity concept the question is: *which kind of similarity?* There are so many different concepts of music similarity (perceptual, structural, melodic, rhythmical, and so on) that is not possible to provide a unique definition.

The variety of segmentations reflects to a large extent the variety of musical similarity concepts, and that is the reason why it is correct to have this parameter here. Nevertheless, as stated in Section 4 the model is rather robust respect to metric changes.

In general, we can just say that the set of segments can be endowed with a notion of distance

$$d : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$$

between pairs of segments and turns this set into a (possibly metric) space  $(\mathcal{S}, d)$ . A natural choice for point sets of a metric space is the Hausdorff metric [13] but any other distance discovered to be useful in music perception, like EMD/PTD [23], can be chosen as well.

Here we assume  $d$  to be:

1. real,
2. non-negative,
3. symmetric and
4. such that  $d(s, s) = 0, \forall s \in \mathcal{S}$

As a matter of fact, most musically relevant perceptual distances do not satisfy all metric axioms [23]. Therefore no further property, like the identity of indiscernibles or the triangle inequality, is assumed.

Given two segments  $s_1$  and  $s_2$ , for the experiments we adopted the two following simple metrics:

$$d_1(s_1, s_2) = \sqrt{\sum_{|s|} |[s_1]_{12} - [s_2]_{12}|} \quad (1)$$

$$d_2(s_1, s_2) = \sqrt{\sum_{|s|} (s'_1 - s'_2)^2} \quad (2)$$

where  $s'$  is the derivative operator on the sequence  $s$ ,  $|s|$  is the length of  $s$  and  $[s]_{12}$  is the sequence  $s$  where each entry has been chosen in the interval  $[0, 11]$ .

$d_1$  is a first-order metric that takes into account just octave transpositions of melodies. In fact, pitch classes out of the range  $[0, 11]$  are folded back into the same interval,

so melodies which differ for one or more octaves belong to the same congruence class modulo 12 semitones.  $d_2$  is a second-order metric that takes into account arbitrary transpositions and inversions of a melody. No other assumptions on possible variations have been made, so that an equivalence class of melodies is composed just of transpositions and inversions of the same melody like in [1].

Both distances can be applied to single voice sequences but also to multiple voice sequences, given that a suitable representation has been provided. For instance, in a two voice piece, with voices  $v_1$  and  $v_2$ , one can consider the difference vector  $v = v_1 - v_2$  as a good representation of a specific segment, and then apply  $d_1$  or  $d_2$  to this new object. The advantage of using this differential representation is that it is invariant respect to transpositions and inversions of the two voices so that, for instance, it makes also  $d_1$  invariant respect to transpositions and inversions, and not just to octave shifts.

By exploiting those distance concepts, it is possible to endow the edges of the complete graph with metric weights in order to compute the weights of nodes in terms of the main eigenvector, as we are going to show in the following Sections.

### 3.3 Matrix representation and ranking eigenvector

The adopted algebraic representation of the ‘score graph’  $K$  is the adjacency matrix  $A(K)$ . This is a nonnegative matrix as its entries are the distance values between the different segments in which the score has been divided into. Perron-Frobenius theory for nonnegative matrices grants that, if  $A \in M_{n \times n}$  and  $A \geq 0$ , then there is an eigenvector  $x \in \mathbb{R}^n$  with  $x \geq 0$  and  $\sum_{i=1}^n x_i = 1$ , called the *Perron vector* of  $A$  [14].

This result has a natural interpretation in the theory of finite Markov chains, where it is the matrix-theoretic equivalent of the convergence of a finite Markov chain, formulated in terms of the transition matrix of the chain [2].

The Perron vector can be viewed as a probability distribution of presence of a ‘random listener’ on a particular segment of a musical piece. This listener recalls with probability  $d(s_i, s_j)$  segment  $s_j$  from segment  $s_i$ , following the ‘links’ represented by the values of the similarity function.

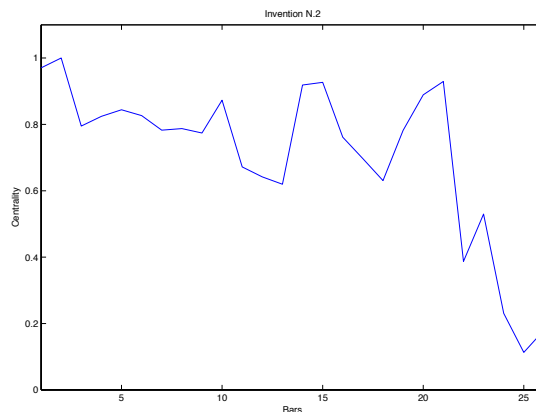
### 3.4 The algorithm

Let  $d : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$  denote a distance function on  $\mathcal{S}$ , like those defined in Section 3.2, which assigns each pair of segments  $s_i$  and  $s_j$  a distance  $d(s_i, s_j)$ . We can describe the algorithm through the following steps:

1. Form the distance matrix  $A = [a_{i,j}]$  such that  $a_{i,j} = d(s_i, s_j)$ ;
2. Form the affinity matrix  $W = [w_{i,j}]$  defined by

$$w_{i,j} = \exp\left(-\frac{a_{i,j}^2}{2\sigma^2}\right) \quad (3)$$

where  $\sigma$  is a parameter that can be chosen experimentally. A possible choice is the standard deviation



**Figure 3.** Normalized eigenvector profile for bars in BWV 773. Higher values correspond to higher centrality (see also Table 2). The metric space is  $(\mathcal{S}, d_1)$ .

tion of the similarity values within the considered network graph;

3. Compute the leading eigenvector  $x = [x_i]$  of  $W$  and rank each segment  $s_i$  according to the component  $x_i$  of  $x$ .

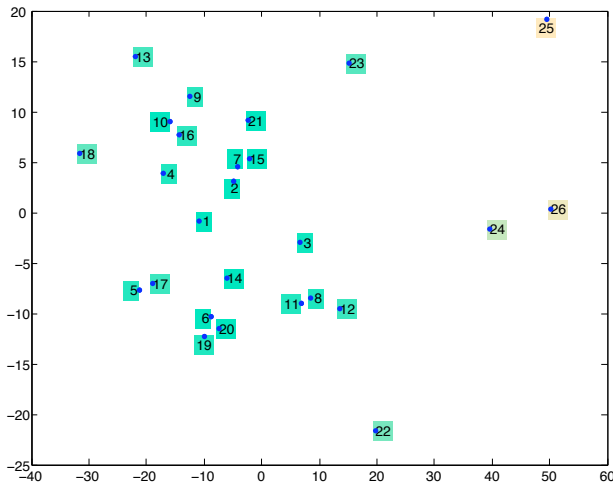
## 4. EXPERIMENTAL RESULTS

In order to evaluate the relevance of the results of the proposed method we need a suitable data collection together with a commonly acceptable ground truth for that collection. Following [1], Johann Sebastian Bach’s *Two-part Inventions* has been our choice. For this collection, a complete ground truth is provided by musicological analysis and it can be found for example in [12] and [24].

The first choice we had to make was the segment size. Many experiments has been conducted but, as stated before, it turned out that reductions of the segment size (for example from two bars to one bar) did not sensibly affect the results. So experiments have been performed with a one-bar long window. Experiments have been performed also to verify the suitability of an overlapping technique but we did not observed any improvement in the results.

Second, we implemented the functional metrics described in Section 3.3. By performing the experiments, we observed a few variations in the first two ranked levels, and this means that top ranked bars tend to be more ‘stable’ respect to metric changes. Thus we can say that the method is rather robust, as far as these metrics are concerned. In the synthesis reported in Table 2 we considered just the top ranked segments, i.e. corresponding to the two (different) highest values of  $x$  components.

When compared to musicological analysis [1] [12] [24] it is evident that the centrality-based model outperforms the repetition-based model, providing also more significative information. Segments with higher rank in the relational model represent always relevant bars of the score, even if they may be different by using different metrics. This means that relevant bars contain a main motif or characterizing sequences. It is not the same for the model based



**Figure 4.** 2D projection of the metric space  $(\mathcal{S}, d_1)$  for BWV 773. Bars with higher centrality values (darker labels) tend to occupy the central region of the graph.

Model	Precision (%)
Repetition	43
$d_1$	77
$d_2$	95

**Table 1.** Precision results for the three models applied to J. S. Bach’s Inventions.

on repetitions: here the relevancy really depends just on the number of repetitions, so it can happen that a trill turns to be more relevant than the rest of the piece just because its repetition rate is higher than that of the other bars.

Bar ranking is in principle not affected by the repetition rate of patterns and higher importance is equally given to higher and lower repetition rates. Of course, superpositions of the two methods may happen too. On the other hand, cases exist for which no repetition occurs and, consequently, the repetition paradigm is not applicable in principle, unless defining ad hoc neighborhood concepts for each piece. In these cases, motif centrality can provide significant results.

In Figure 3 the components of the main eigenvector for BWV 773, representing the degree of centrality of each bar, have been plotted against bar numbers. This provides an immediate representation of the “importance” of each bar within the whole piece. Bars with higher values are more likely to contain a main motif of the piece. In particular, for BWV 773, bars 1 and 2 actually contain the main motif.

Figure 4 shows a two-dimensional projection of the 26-dimensional metric space for BWV 773 obtained through a dimensionality reduction algorithm. From this picture it is evident how the top ranked results (1, 2) occupy the central region of the graph and have darker labels, as the darkness is directly proportional to the correspondent component of the main eigenvector, and thus to the centrality, in the sense of graph theory, of the correspondent segment.

Table 1 presents a synthesis of the results shown in Table 2 in terms of the precision of the three methods. As for the computational complexity, suitable linear eigensolvers are available, and they can be easily applied, especially in case of very long pieces.

## 5. CONCLUSIONS

We presented a new approach for motif discovery in music pieces based on an eigenvector method. Scores are segmented into a network of bars and then ranked depending on their graph centrality. Bars with higher centrality are more likely to be musically relevant and can be exploited for music summarization. Experiments performed on the collection of J.S.Bach’s 2-parts Inventions show the effectiveness of our method.

Besides music information retrieval, we expect this approach to find applications in music theory, perception and visualization. For instance, one could investigate how particular mathematical entities (e.g. spectra) relate to particular musical issues (e.g. genre, authorship).

Second, one could investigate how different metrics  $d$  relate to different concepts of melodic and harmonic similarity; in this context, the inverse problem of finding metrics  $d$  induced by a priori eigenvectors (coming from a hand-made musicological analysis) could provide interesting insights into music similarity perception.

Third, it is also possible to compare different music pieces from a structural point of view by comparing their associated eigenvectors.

Finally, the method could be extended to the audio domain, for instance to organize large audio collections, where heuristic methods can be hardly applied and it is usually difficult or even impossible to separate different voices and/or musical instruments.

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$N^\circ$	Catalog	Repeated bars	$d_1$	$d_2$
1	BWV 772	$\emptyset$	16, 18, 17, 3	16, 18
2	BWV 773	2, 3, 24, 25	2, 1, 21, 15	3, 25, 2, 24
3	BWV 774	2, 20, 42, 50, 55	54, 41, 19, 48	2, 20, 42, 50, 55
4	BWV 775	1, 2, 5, 6, 19, 20, 21, 44, 45	6, 2, 29, 22	1, 5, 44, 2, 6, 45
5	BWV 776	2, 3, 28, 29	2, 16, 17, 27	2, 28, 3, 29
6	BWV 777	5, 25, 42, 63, 84, 105	56, 21, 6, 20	5, 25, 42, 63, 84, 105
7	BWV 778	$\emptyset$	10, 8, 5, 1	7, 4
8	BWV 779	$\emptyset$	1, 6, 20	6, 26
9	BWV 780	1, 2, 3, 4, 29, 30, 31, 32	31, 14, 1, 2	1, 3, 29, 2
10	BWV 781	20, 21, 22, 23, 27, 29, 31	1, 17, 27, 32	1, 27, 29, 31
11	BWV 782	$\emptyset$	17, 4, 14, 6	6, 17
12	BWV 783	$\emptyset$	5, 6, 17, 16	9, 5
13	BWV 784	$\emptyset$	18, 2, 1, 21	8, 1
14	BWV 785	12, 14	8, 6, 7, 16	12, 14, 16
15	BWV 786	$\emptyset$	7, 10, 14, 9	12, 4

**Table 2.** Experimental results for the repetition paradigm (using both metrics  $d_1$  and  $d_2$ ) and the relational paradigm. Gray numbers represents irrelevant bars.

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